

Multistage rocket

Assume: n stages, exhaust speed u

each stage of rocket has same mass ratio at burnout, $k = \frac{m_i}{m_f}$

To show: final speed at n^{th} stage is $nu \ln k$

This is really just a slight extension of the arguments that lead to eqs 9.153 and 9.154.

After one stage 9.153 gives
second stage

$$V_1 = v_0 + u \ln k$$

$$V_2 = v_1 + u \ln k \\ = v_0 + 2u \ln k$$

third

$$V_3 = v_2 + u \ln k = v_0 + 3u \ln k$$

n^{th} stage:

$$\underline{V_n = v_0 + nu \ln k} \quad \checkmark$$