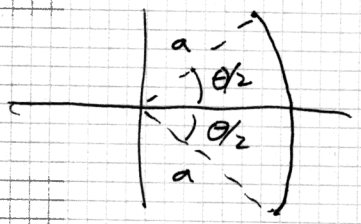
TM 9.4



Assume mass per unit length λ

- total mass $M = \lambda l = \lambda a\theta$
- $\vec{R} = \frac{1}{M}\int \vec{r}\, dm = \frac{1}{M}\int_{-\theta/2}^{\theta/2} \vec{r}\, \lambda a\, d\phi$

$dm = \lambda dl$

$= \lambda a\, d\phi$

$$R_x = \frac{\lambda a}{M}\int_{-\theta/2}^{\theta/2} a\cos\phi\, d\phi = \frac{\lambda a^2}{M}\sin\phi\Big|_{-\theta/2}^{\theta/2}$$

$$= \frac{\lambda a^2}{M}\, 2\sin\theta/2 = (\lambda a^2)\frac{1}{\lambda a\theta}\, 2\sin\frac{\theta}{2} = \frac{2a}{\theta}\sin\frac{\theta}{2}$$

$$R_y = \frac{\lambda a}{M}\int_{-\theta/2}^{\theta/2} a\sin\phi\, d\phi = \frac{\lambda a^2}{M}(-\cos\phi)\Big|_{-\theta/2}^{\theta/2}$$

$= 0$, as we know from symmetry.

So: Center of Mass is at $\left(\frac{2a}{\theta}\sin\frac{\theta}{2}, 0\right)$