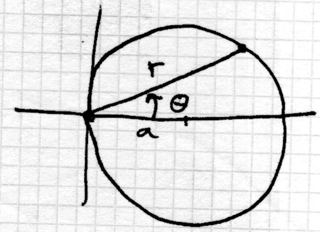
TM 8.11

- The orbit is a circle osculating origin.
  We can write it as

$$r = 2a\cos\Theta$$



- Using eq. 8.20

  - $$u = \frac{1}{r} = \frac{1}{2a\cos\theta}$$

  - $$\frac{d^2u}{d\Theta^2} + u = -\frac{\mu}{\ell^2}\frac{F}{u^2}$$

- By assumption, $F = -ku^{+n} = -\frac{k}{r^n}$

  so: $$u'' + u = +\frac{k\mu}{\ell^2}u^{n-2}$$

- Now substitute our expression for the orbit eqn. into the differential eqn an see if it's satisfied for any values of n (hopefully, for n=5)

$$u = \frac{1}{2a}(\cos\theta)^{-1}$$

$$u' = -\frac{1}{2a}(\cos\theta)^{-2}(-\sin\theta) = \frac{\sin\theta}{\cos^2\theta}\frac{1}{2a}$$

$$u'' = \frac{1}{2a}\frac{d}{d\theta}(\sin\theta\cos^{-2}\theta) = \frac{1}{2a}\left[\cos\theta\cos^{-2}\theta + \sin\theta(-2)(\cos\theta)^{-3}(-\sin\theta)\right]$$

$$= \frac{1}{2a}\left[\cos^{-1}\theta + 2\sin^2\theta\cos^{-3}\theta\right]$$

$$= \frac{1}{2a}\frac{1}{\cos^3\theta}\left[\cos^2\theta + 2\sin^2\theta\right] = \frac{1}{2a}\frac{1}{\cos^3\theta}\left[\cos^2\theta + 2(1-\cos^2\theta)\right]$$

$$= \frac{1}{2a}\frac{1}{\cos^3\theta}\left[2 - \cos^2\theta\right]$$

Eqn is:

$$u'' + u - \frac{k\mu}{\ell^2}u^{n-2} = 0$$

so:

$$\frac{1}{2a}\frac{1}{\cos^3\theta}\left[2-\cos^2\theta\right] + \frac{1}{2a}\frac{1}{\cos\theta} - \frac{k\mu}{\ell^2}\frac{1}{(2a)^{n-2}}\frac{1}{(\cos\theta)^{n-2}} = 0$$

$$\frac{2}{\cos^3\theta} - \cancel{\frac{1}{\cos\theta}} + \cancel{\frac{1}{\cos\theta}} - \frac{k\mu}{\ell^2}\frac{1}{(2a)^{n-1}}\frac{1}{(\cos\theta)^{n-2}} = 0$$

$$\frac{2}{\cos^3\theta} = \frac{k\mu}{\ell^2}\frac{1}{(2a)^{n-1}}\frac{1}{(\cos\theta)^{n-2}}$$

For $n=5$: $\cos^3\theta$ appear in denom on both sides. Cancel, leaving

$$2 = \frac{k\mu}{\ell^2}\frac{1}{(2a)^4} \longrightarrow k = \frac{2\ell^2}{\mu}(2a)^4$$