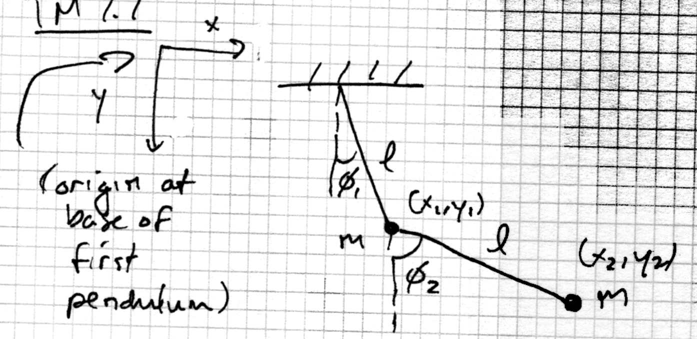
TM 7.7



Position of upper mass: $\begin{cases} x_1 = \ell \sin\phi_1 \\ y_1 = \ell \cos\phi_1 \end{cases}$

Position of lower mass: $\begin{cases} x_2 = \ell(\sin\phi_1 + \sin\phi_2) \\ y_2 = \ell(\cos\phi_1 + \cos\phi_2) \end{cases}$

$$L = T_1 + T_2 - (U_1 + U_2)$$

$$T_1 = \tfrac{1}{2} m (\dot{x}_1^2 + \dot{y}_1^2)$$
$$= \tfrac{1}{2} m \ell^2 ((\cos\phi_1 \dot{\phi}_1)^2 + (-\sin\phi_1 \dot{\phi}_1)^2)$$
$$= \tfrac{1}{2} m \ell^2 \dot{\phi}_1^2$$

$$T_2 = \tfrac{1}{2} m (\dot{x}_2^2 + \dot{y}_2^2)$$
$$= \tfrac{1}{2} m \ell^2 ((\cos\phi_1 \dot{\phi}_1 + \cos\phi_2 \dot{\phi}_2)^2 + (-\sin\phi_1 \dot{\phi}_1 - \sin\phi_2 \dot{\phi}_2)^2)$$
$$= \tfrac{1}{2} m \ell^2 [(\cos^2\phi_1 + \sin^2\phi_1)\dot{\phi}_1^2 + (\cos^2\phi_2 + \sin^2\phi_2)\dot{\phi}_2^2 + 2\cos\phi_1 \cos\phi_2 \dot{\phi}_1 \dot{\phi}_2 + 2\sin\phi_1 \sin\phi_2 \dot{\phi}_1 \dot{\phi}_2]$$
$$= \tfrac{1}{2} m \ell^2 [\dot{\phi}_1^2 + \dot{\phi}_2^2 + 2(\cos\phi_1 \cos\phi_2 + \sin\phi_1 \sin\phi_2)\dot{\phi}_1 \dot{\phi}_2]$$
$$= \tfrac{1}{2} m \ell^2 [\dot{\phi}_1^2 + \dot{\phi}_2^2 + 2\cos(\phi_1 - \phi_2)\dot{\phi}_1 \dot{\phi}_2]$$

$$U_1 = -mgy_1 = -mg\ell\cos\phi_1$$
$$U_2 = -mgy_2 = -mg\ell(\cos\phi_1 + \cos\phi_2)$$

$$L = \tfrac{1}{2} m\ell^2 [\dot{\phi}_1^2 + \dot{\phi}_2^2 + 2\cos(\phi_1 - \phi_2)\dot{\phi}_1 \dot{\phi}_2] + mg\ell[2\cos\phi_1 + \cos\phi_2]$$

Lagrange's eqns?

for ϕ_1: $$\frac{\partial L}{\partial \phi_1} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}_1} = 0$$

$$\frac{\partial L}{\partial \phi_1} = -m\ell^2 \sin(\phi_1 - \phi_2)\dot{\phi}_1 \dot{\phi}_2 - 2mg\ell \sin\phi_1$$

$$\frac{\partial L}{\partial \dot{\phi}_1} = \tfrac{1}{2} m\ell^2 [2\dot{\phi}_1 + 2\cos(\phi_1 - \phi_2)\dot{\phi}_2]$$

$$\frac{\partial L}{\partial \phi_1} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}_1} = 0$$

$$0 = -m\ell^2 \sin(\phi_1 - \phi_2)\dot{\phi}_1 \dot{\phi}_2 - 2mg\ell\sin\phi_1 - \tfrac{1}{2} m\ell^2 \frac{d}{dt}[2\dot{\phi}_1 + 2\cos(\phi_1 - \phi_2)\dot{\phi}_2]$$

$$0 = \sin(\phi_1 - \phi_2)\dot{\phi}_1 \dot{\phi}_2 + \frac{2g}{\ell}\sin\phi_1 + \frac{d}{dt}[\dot{\phi}_1 + \cos(\phi_1 - \phi_2)\dot{\phi}_2]$$

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$$0 = \sin(\phi_1-\phi_2)\,\dot\phi_1\dot\phi_2 + \frac{2g}{l}\sin\phi_1 + \left[\ddot\phi_1 + \ddot\phi_2\cos(\phi_1-\phi_2) - \sin(\phi_1-\phi_2)\,\dot\phi_2\,(\dot\phi_1-\dot\phi_2)\right]$$

$$0 = \frac{2g}{l}\sin\phi_1 + \ddot\phi_1 + \ddot\phi_2\cos(\phi_1-\phi_2) + \dot\phi_2^2\sin(\phi_1-\phi_2)$$

$$\boxed{0 = \ddot\phi_1 + \ddot\phi_2\cos(\phi_1-\phi_2) + \dot\phi_2^2\sin(\phi_1-\phi_2) + \frac{2g}{l}\sin\phi_1}$$

For ϕ_2:

$$\frac{\partial L}{\partial \phi_2} - \frac{d}{dt}\frac{\partial L}{\partial \dot\phi_2} = 0$$

$$\frac{\partial L}{\partial \phi_2} = ml^2(+\sin(\phi_1-\phi_2))\,\dot\phi_2\,\dot\phi_1 - mgl\sin\phi_2$$

$$\frac{\partial L}{\partial \dot\phi_2} = ml^2\dot\phi_2 + \cos(\phi_1-\phi_2)\,\dot\phi_1\, ml^2$$

$$0 = ml^2\sin(\phi_1-\phi_2)\,\dot\phi_1\dot\phi_2 - mgl\sin\phi_2 - \frac{d}{dt}\left[ml^2\dot\phi_2 + ml^2\dot\phi_1\cos(\phi_1-\phi_2)\right]$$

$$0 = \sin(\phi_1-\phi_2)\,\dot\phi_1\dot\phi_2 - \frac{g}{l}\sin\phi_2 - \left[\ddot\phi_2 + \ddot\phi_1\cos(\phi_1-\phi_2) + \dot\phi_1(-\sin(\phi_1-\phi_2))(\dot\phi_1-\dot\phi_2)\right]$$

$$0 = -\frac{g}{l}\sin\phi_2 - \ddot\phi_2 - \ddot\phi_1\cos(\phi_1-\phi_2) + \dot\phi_1^2\sin(\phi_1-\phi_2)$$

$$\boxed{0 = \ddot\phi_2 + \ddot\phi_1\cos(\phi_1-\phi_2) - \dot\phi_1^2\sin(\phi_1-\phi_2) + \frac{g}{l}\sin\phi_2}$$

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