$M = 2m$

Position of cart: $x_c$

Position of mass 1: $(x_1, y_1) =$
$$\begin{cases} x_1 = x_c + b\sin\phi_1 \\ y_1 = b\cos\phi_1 \end{cases}$$

Position of mass 2: $(x_2, y_2) =$
$$\begin{cases} x_2 = x_1 + b\sin\phi_2 \\ \quad = x_c + b(\sin\phi_1 + \sin\phi_2) \\ y_2 = y_1 + b\cos\phi_2 = b(\cos\phi_1 + \cos\phi_2) \end{cases}$$

- $T_{cart} = \frac{1}{2}M\dot{x}_c^2 = m\dot{x}_c^2$
- $T_{mass\,1} = \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2)$

$$\begin{cases} \dot{x}_1 = \dot{x}_c + b\cos\phi_1\,\dot{\phi}_1 \\ \dot{y}_1 = -b\sin\phi_1\,\dot{\phi}_1 \end{cases}$$

$$\begin{cases} \dot{x}_1^2 = (\dot{x}_c + b\cos\phi_1\,\dot{\phi}_1)^2 = \dot{x}_c^2 + (b\dot{\phi}_1)^2\cos^2\phi_1 + 2\dot{x}_c\,b\dot{\phi}_1\cos\phi_1 \\ \dot{y}_1^2 = (-b\sin\phi_1\,\dot{\phi}_1)^2 = (b\dot{\phi}_1)^2\sin^2\phi_1 \end{cases}$$

$$\dot{x}_1^2 + \dot{y}_1^2 = \dot{x}_c^2 + (b\dot{\phi}_1)^2 + 2\dot{x}_c(b\dot{\phi}_1)\cos\phi_1$$

$$T_{mass\,1} = \frac{1}{2}m\left[\dot{x}_c^2 + b^2\dot{\phi}_1^2 + 2\dot{x}_c\,b\dot{\phi}_1\cos\phi_1\right]$$

- $T_{mass\,2} = \frac{1}{2}m(\dot{x}_2^2 + \dot{y}_2^2)$

$$\dot{x}_2 = \dot{x}_c + b(\cos\phi_1\,\dot{\phi}_1 + \cos\phi_2\,\dot{\phi}_2)$$
$$\dot{y}_2 = -b(\sin\phi_1\,\dot{\phi}_1 + \sin\phi_2\,\dot{\phi}_2)$$

$$\dot{x}_2^2 = \dot{x}_c^2 + b^2(\cos\phi_1\,\dot{\phi}_1 + \cos\phi_2\,\dot{\phi}_2)^2 + 2\dot{x}_c\,b(\cos\phi_1\,\dot{\phi}_1 + \cos\phi_2\,\dot{\phi}_2)$$
$$= \dot{x}_c^2 + b^2\left[\cos^2\phi_1\,\dot{\phi}_1^2 + \cos^2\phi_2\,\dot{\phi}_2^2 + 2\dot{\phi}_1\,\dot{\phi}_2\cos\phi_1\cos\phi_2\right]$$
$$\qquad + 2\dot{x}_c\,b(\cos\phi_1\,\dot{\phi}_1 + \cos\phi_2\,\dot{\phi}_2)$$

$$\dot{y}_2^2 = b^2\left(\sin^2\phi_1\,\dot{\phi}_1^2 + \sin^2\phi_2\,\dot{\phi}_2^2 + 2\sin\phi_1\sin\phi_2\,\dot{\phi}_1\,\dot{\phi}_2\right)$$

$$\dot{x}_2^2 + \dot{y}_2^2 = \dot{x}_c^2 + b^2\dot{\phi}_1^2(\cos^2\phi_1 + \sin^2\phi_1) + b^2\dot{\phi}_2^2(\cos^2\phi_2 + \sin^2\phi_2)$$
$$\qquad + 2b^2\dot{\phi}_1\dot{\phi}_2(\cos\phi_1\cos\phi_2 + \sin\phi_1\sin\phi_2)$$
$$\qquad + 2\dot{x}_c\,b(\cos\phi_1\,\dot{\phi}_1 + \cos\phi_2\,\dot{\phi}_2)$$
$$= \dot{x}_c^2 + b^2(\dot{\phi}_1^2 + \dot{\phi}_2^2) + 2b^2\dot{\phi}_1\dot{\phi}_2\cos(\phi_1 - \phi_2)$$
$$\qquad + 2\dot{x}_c\,b(\cos\phi_1\,\dot{\phi}_1 + \cos\phi_2\,\dot{\phi}_2)$$

- $U_{mass\,1} = -mgy_1 = -mgb\cos\phi_1$
- $U_{mass\,2} = -mgy_2 = -mgb(\cos\phi_1 + \cos\phi_2)$

- $L = (T_{mass\,1} + T_{mass\,2} + T_{cart}) - (U_{mass\,1} + U_{mass\,2})$

$$L = m\dot{x}_c^2 + \tfrac{1}{2}m[\dot{x}_c^2 + b^2\dot{\phi}_1^2 + 2\dot{x}_c b\dot{\phi}_1\cos\phi_1]$$
$$+ \tfrac{1}{2}m[\dot{x}_c^2 + b^2(\dot{\phi}_1^2 + \dot{\phi}_2^2) + 2b^2\dot{\phi}_1\dot{\phi}_2\cos(\phi_1-\phi_2) + 2\dot{x}_c b(\cos\phi_1\dot{\phi}_1 + \cos\phi_2\dot{\phi}_2)]$$
$$+ mgb\cos\phi_1 + mgb(\cos\phi_1 + \cos\phi_2)$$

$$L = 2m\dot{x}_c^2 + mb^2\dot{\phi}_1^2 + \tfrac{1}{2}mb\dot{\phi}_2^2 + 2\dot{x}_c b\dot{\phi}_1\cos\phi_1 m + m\dot{x}_c b\dot{\phi}_2\cos\phi_2$$
$$+ mb^2\dot{\phi}_1\dot{\phi}_2\cos(\phi_1-\phi_2) + 2mgb\cos\phi_1 + mgb\cos\phi_2$$

Lagrange eqns for $x_c, \phi_1, \phi_2$:

① $\quad \dfrac{\partial L}{\partial x_c} - \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{x}_c} = 0$

$$\dfrac{\partial L}{\partial x_c} = 0$$

$$\dfrac{\partial L}{\partial \dot{x}_c} = m\dot{x}_c + 2b\dot{\phi}_1\cos\phi_1 m + mb\dot{\phi}_2\cos\phi_2$$

$$\boxed{m\dfrac{d}{dt}\left[\dot{x}_c + 2b\dot{\phi}_1\cos\phi_1 + b\dot{\phi}_2\cos\phi_2\right] = 0}$$

② $\quad \dfrac{\partial L}{\partial \phi_1} - \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{\phi}_1} = 0$

$$\dfrac{\partial L}{\partial \phi_1} = -2\dot{x}_c b\dot{\phi}_1\sin\phi_1 m - mb^2\dot{\phi}_1\dot{\phi}_2\sin(\phi_1-\phi_2) - 2mbg\sin\phi_1$$

$$\dfrac{\partial L}{\partial \dot{\phi}_1} = 2mb^2\dot{\phi}_1 + 2\dot{x}_c bm\cos\phi_1 + mb^2\dot{\phi}_2\cos(\phi_1-\phi_2)$$

$$\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\phi}_1}\right) = mb\dfrac{d}{dt}\left[2b\dot{\phi}_1 + 2\dot{x}_c\cos\phi_1 + b\dot{\phi}_2\cos(\phi_1-\phi_2)\right]$$

$$-2\dot{x}_c b\dot{\phi}_1\sin\phi_1 m - mb^2\dot{\phi}_1\dot{\phi}_2\sin(\phi_1-\phi_2) - 2mbg\sin\phi_1 =$$
$$mb\dfrac{d}{dt}\left[2b\dot{\phi}_1 + 2\dot{x}_c\cos\phi_1 + b\dot{\phi}_2\cos(\phi_1-\phi_2)\right]$$

$$\boxed{-\left[2\dot{x}_c\dot{\phi}_1\sin\phi_1 + b\dot{\phi}_1\dot{\phi}_2\sin(\phi_1-\phi_2) + 2g\sin\phi_1\right] = \dfrac{d}{dt}\left[2b\dot{\phi}_1 + 2\dot{x}_c\cos\phi_1 + b\dot{\phi}_2\cos(\phi_1-\phi_2)\right]}$$

③ $\quad \dfrac{\partial L}{\partial \phi_2} - \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{\phi}_2} = 0$

$$\dfrac{\partial L}{\partial \phi_2} = -m\dot{x}_c b\dot{\phi}_2\sin\phi_2 + mb^2\dot{\phi}_1\dot{\phi}_2\sin(\phi_1-\phi_2) - mgb\sin\phi_2$$

$$\dfrac{\partial L}{\partial \dot{\phi}_2} = mb\dot{\phi}_2 + m\dot{x}_c b\cos\phi_2 + mb^2\dot{\phi}_1\cos(\phi_1-\phi_2)$$

$$\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\phi}_2}\right) = mb\dfrac{d}{dt}\left[b\dot{\phi}_2 + \dot{x}_c\cos\phi_2 + b\dot{\phi}_1\cos(\phi_1-\phi_2)\right]$$

$$\boxed{-\dot{x}_c\dot{\phi}_2\sin\phi_2 + b\dot{\phi}_1\dot{\phi}_2\sin(\phi_1-\phi_2) - g\sin\phi_2 = \dfrac{d}{dt}\left[b\dot{\phi}_2 + \dot{x}_c\cos\phi_2 + b\dot{\phi}_1\cos(\phi_1-\phi_2)\right]}$$

Simplifying a bit:

① $\boxed{\dot{x}_c + 2b\dot{\phi}_1\cos\phi_1 + b\dot{\phi}_2\cos\phi_2 = \text{const}}$

② $-2\ddot{x}_c\,\dot{\phi}_1\sin\phi_1 - b\dot{\phi}_1\dot{\phi}_2\sin(\phi_1-\phi_2) - 2g\sin\phi_1 =$

$= 2b\ddot{\phi}_1 + 2\ddot{x}_c\cos\phi_1 - 2\dot{x}_c\dot{\phi}_1\sin\phi_1 + b\ddot{\phi}_2\cos(\phi_1-\phi_2)$

$\qquad - b\dot{\phi}_2\sin(\phi_1-\phi_2)(\dot{\phi}_1-\dot{\phi}_2)$

$\boxed{-2g\sin\phi_1 = 2b\ddot{\phi}_1 + 2\ddot{x}_c\cos\phi_1 + b\ddot{\phi}_2\cos(\phi_1-\phi_2) + b\dot{\phi}_2^2\sin(\phi_1-\phi_2)}$

③ $-\dot{x}_c\dot{\phi}_2\sin\phi_2 + b\dot{\phi}_1\dot{\phi}_2\sin(\phi_1-\phi_2) - g\sin\phi_2 =$

$b\ddot{\phi}_2 + \ddot{x}_c\cos\phi_2 - \dot{x}_c\dot{\phi}_2\sin\phi_2 + b\ddot{\phi}_1\cos(\phi_1-\phi_2) - b\dot{\phi}_1\sin(\phi_1-\phi_2)(\dot{\phi}_1-\dot{\phi}_2)$

$\boxed{-g\sin\phi_2 = b\ddot{\phi}_2 + \ddot{x}_c\cos\phi_2 + b\ddot{\phi}_1\cos(\phi_1-\phi_2) - b\dot{\phi}_1^2\sin(\phi_1-\phi_2)}$