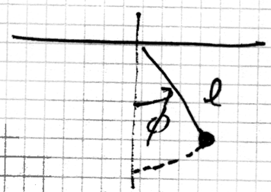
(a) Simple pendulum



$$T = \frac{1}{2}m(l\dot{\phi})^2$$

$$U = mgl(1-\cos\phi)$$

- $L = T - U = \frac{1}{2}ml^2\dot{\phi}^2 - mgl(1-\cos\phi)$
- Find p_ϕ: $p_\phi = \frac{\partial L}{\partial\dot{\phi}} = ml^2\dot{\phi} \Longrightarrow \dot{\phi} = \frac{p_\phi}{ml^2}$
- $H = p_\phi\dot{\phi} - L$

$$= p_\phi\dot{\phi} - \frac{1}{2}ml^2\dot{\phi}^2 + mgl(1-\cos\phi)$$

$$= \frac{p_\phi^2}{m} - \frac{1}{2}ml^2\left(\frac{p_\phi}{ml}\right)^2 + mgl(1-\cos\phi)$$

$$H = \frac{p_\phi^2}{2m} + mgl(1-\cos\phi)$$

- Hamilton's equations of motion

① $\frac{\partial H}{\partial p_\phi} = \dot{\phi}$

$\frac{p_\phi}{m} = \dot{\phi}$ ⟶ this reproduces the expression for p_ϕ above

② $\frac{\partial H}{\partial \phi} = -\dot{p}_\phi$

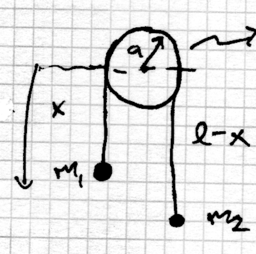
$$mgl\sin\phi = -\dot{p}_\phi$$

$$mgl\sin\phi = -ml^2\ddot{\phi}$$

$$\boxed{\ddot{\phi} + \frac{g}{l}\sin\phi = 0}$$

Newton's 2nd law for simple pendulum

(b) Atwood Machine



Assume a pulley of radius a, moment of inertia I

so: $T_1 = \frac{1}{2}m_1\dot{x}^2$

$T_2 = \frac{1}{2}m_2\dot{y}^2 = \frac{1}{2}m_2\dot{x}^2$

$T_p = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{\dot{x}}{a}\right)^2$

$\Big\} \rightarrow T = \frac{1}{2}(m_1+m_2)\dot{x}^2 + \frac{1}{2}I\left(\frac{\dot{x}}{a}\right)^2$

$U = -m_1gx - m_2g(l-x)$ → drop it

$= (m_2 - m_1)gx + \text{const.}$

Write Lagrangian: $L = T - U = \frac{1}{2}(m_1+m_2)\dot{x}^2 + \frac{1}{2}I\frac{\dot{x}^2}{a^2} - (m_2-m_1)gx$

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Find canonical mom.: (2)

- $p = \dfrac{\partial L}{\partial \dot{x}} = \left(m_1 + m_2 + \dfrac{I}{a^2}\right)\dot{x} \implies \dot{x} = \dfrac{p}{m_1 + m_2 + \frac{I}{a^2}}$

Form Hamiltonian fn.:

- $$\begin{aligned} H &= p\dot{x} - L \\ &= p\dot{x} - \tfrac{1}{2}\left[m_1 + m_2 + \tfrac{I}{a^2}\right]\dot{x}^2 + (m_2 - m_1)gx \\ &= \frac{p^2}{2\left(m_1 + m_2 + \frac{I}{a^2}\right)} + (m_2 - m_1)gx \end{aligned}$$

Hamilton's eqns. of Motion

① $\dfrac{\partial H}{\partial p} = \dot{x}$

$$\boxed{\frac{p}{m_1 + m_2 + \frac{I}{a^2}} = \dot{x}}$$

$\Longrightarrow$ confirms our def. of p, as above

② $\dfrac{\partial H}{\partial x} = -\dot{p}$

$$\boxed{(m_2 - m_1)g = -\dot{p}}$$

so: $-(m_2 - m_1)g = \left(m_1 + m_2 + \dfrac{I}{a^2}\right)\ddot{x}$

$$\boxed{\ddot{x} = \frac{-(m_2 - m_1)g}{\left(m_1 + m_2 + \frac{I}{a^2}\right)}}$$

just as Newton II gives for a simple Atwood machine