TM 7-23

Particle moving freely in a conservative force field whose potential function is U.

- $T = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$
- $L = T - U = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - U$
- The canonical momenta are:

$$\begin{cases} p_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x} \Rightarrow \dot{x} = \frac{p_x}{m} \\ p_y = \frac{\partial L}{\partial \dot{y}} = m\dot{y} \Rightarrow \dot{y} = \frac{p_y}{m} \\ p_z = \frac{\partial L}{\partial \dot{z}} = m\dot{z} \Rightarrow \dot{z} = \frac{p_z}{m} \end{cases}$$

- The Hamiltonian function is

$$\begin{aligned} H &= (p_x\dot{x} + p_y\dot{y} + p_z\dot{z}) - L \\ &= (p_x\dot{x} + p_y\dot{y} + p_z\dot{z}) - \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + U \\ &= \frac{(p_x^2 + p_y^2 + p_z^2)}{2m} + U \end{aligned}$$

- Hamilton's equations of motion are:

$$\begin{cases} ① \ \dot{q}_i = \frac{\partial H}{\partial p_i} \\ ② \ -\dot{p}_i = \frac{\partial H}{\partial q_i} \end{cases}$$

For x: ① $\dot{x} = \frac{\partial H}{\partial p_x} = \frac{p_x}{2m}$ $\longrightarrow$ just returns the definition of p_x from above

② $\boxed{-\dot{p}_x = \frac{\partial U}{\partial x}}$ $\Longrightarrow$ this is the x-component of Newton II ($\dot{\vec{p}} = -\vec{\nabla}U$)

Similarly for y, z

$$\boxed{\begin{aligned} -\dot{p}_y &= \frac{\partial U}{\partial y} \\ -\dot{p}_z &= \frac{\partial U}{\partial z} \end{aligned}}$$

so, $\frac{d\vec{p}}{dt} = -\vec{\nabla}U$, same as Newton II.