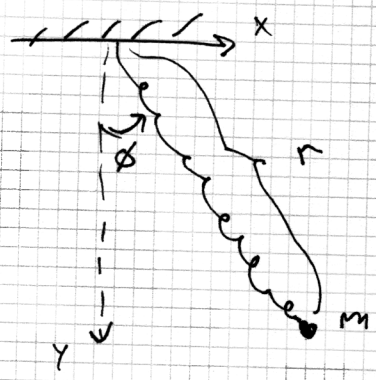
TM 7-15



$L = T - U$

$T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2)$

$\begin{cases} x = r\sin\phi \\ y = r\cos\phi \end{cases}$

$\begin{cases} \dot{x} = \dot{r}\sin\phi + r\cos\phi\,\dot{\phi} \\ \dot{y} = \dot{r}\cos\phi - r\sin\phi\,\dot{\phi} \end{cases}$

$$\begin{aligned} \dot{x}^2 + \dot{y}^2 &= (\dot{r}\sin\phi + r\cos\phi\,\dot{\phi})^2 + (\dot{r}\cos\phi - r\sin\phi\,\dot{\phi})^2 \\ &= \dot{r}^2(\sin^2\phi + \cos^2\phi) + (r\dot{\phi})^2(\cos^2\phi + \sin^2\phi) \\ &\quad + 2r\dot{r}\dot{\phi}(\sin\phi\cos\phi - \sin\phi\cos\phi) \\ &= \dot{r}^2 + (r\dot{\phi})^2 \end{aligned}$$

$T = \frac{1}{2} m(\dot{r}^2 + (r\dot{\phi})^2)$ [could really have written this down directly!]

$U_{sp.} = \frac{1}{2} k(r-b)^2$

$U_{grav} = -mgy = -mgr\cos\phi$

$L = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\phi}^2) - \frac{1}{2} k(r-b)^2 + mgr\cos\phi$

Lagrange eqns?

for r: $\frac{\partial L}{\partial r} - \frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = 0$

$\frac{\partial L}{\partial r} = mr\dot{\phi}^2 - k(r-b) + mg\cos\phi$

$\frac{\partial L}{\partial \dot{r}} = m\dot{r}$

So: $mr\dot{\phi}^2 - k(r-b) + mg\cos\phi = m\ddot{r}$

or $\boxed{\ddot{r} = -\frac{k}{m}(r-b) + g\cos\phi + r\dot{\phi}^2}$

for φ: $\frac{\partial L}{\partial \phi} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} = 0$

$\frac{\partial L}{\partial \phi} = -mgr\sin\phi$

$\frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}$

So: $-mgr\sin\phi = \frac{d}{dt}(mr^2\dot{\phi})$, or $\boxed{-g\sin\phi = 2\dot{r}\dot{\phi} + r\ddot{\phi}}$