

11-28. From Fig. 11-7c we see that $\omega_\theta = \dot{\phi}$ is along the x'_3 -axis, $\omega_\theta = \dot{\theta}$ is along the line of nodes, and $\omega_\psi = \dot{\psi}$ is along the x_3 -axis. Then,

$$\omega'_\theta = \dot{\phi} \mathbf{e}'_3 \quad (1)$$

where \mathbf{e}'_3 is the unit vector in the x'_3 direction.

Projecting the line of nodes into the x'_1 - and x'_2 -axes, we obtain

$$\omega'_\theta = \dot{\theta}(\mathbf{e}'_1 \cos \phi + \mathbf{e}'_2 \sin \phi) \quad (2)$$

ω'_ψ has components along all three of the x'_i axes. First, we write ω'_ψ in terms of a component along the x'_3 -axis and a component normal to this axis:

$$\omega'_\psi = \dot{\psi}(\mathbf{e}'_{12} \sin \theta + \mathbf{e}'_3 \cos \theta) \quad (3)$$

where

$$\mathbf{e}'_{12} = \mathbf{e}'_1 \sin \phi - \mathbf{e}'_2 \cos \phi \quad (4)$$

Then,

$$\omega'_\psi = \dot{\psi}(\mathbf{e}'_1 \sin \theta \sin \phi - \mathbf{e}'_2 \sin \theta \cos \phi + \mathbf{e}'_3 \cos \theta) \quad (5)$$

Collecting the various components, we have

$\begin{aligned} \omega'_1 &= \dot{\theta} \cos \phi + \dot{\psi} \sin \theta \sin \phi \\ \omega'_2 &= \dot{\theta} \sin \phi - \dot{\psi} \sin \theta \cos \phi \\ \omega'_3 &= \dot{\psi} \cos \theta + \dot{\phi} \end{aligned}$
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