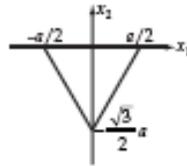


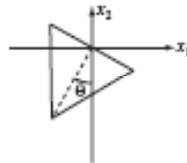
11-24.



The area of the triangle is $A = \sqrt{3} a^2/4$, so that the density is

$$\rho = \frac{M}{A} = \frac{4M}{\sqrt{3} a^2} \quad (1)$$

a) The rotational inertia with respect to an axis through the point of suspension (the origin) is



$$\begin{aligned} I_3 &= \rho \int (x_1^2 + x_2^2) dx_1 dx_2 \\ &= 2\rho \int_0^{a/2} dx_1 \int_{-\frac{\sqrt{3}}{2}(a-2x_1)}^0 (x_1^2 + x_2^2) dx_2 \\ &= \frac{\sqrt{3}}{24} \rho a^4 = \frac{1}{6} M a^2 \end{aligned} \quad (2)$$

When the triangle is suspended as shown and when $\theta = 0$, the coordinates of the center of mass are $(0, \bar{x}_2, 0)$, where

$$\begin{aligned}
 x_2 &= \frac{1}{M} \int \rho x_2 dx_1 dx_2 \\
 &= \frac{2\rho}{M} \int_0^{a/2} dx_1 \int_{-\sqrt{3}(a-2x_1)}^0 x_2 dx_2 \\
 &= -\frac{a}{2\sqrt{3}}
 \end{aligned} \tag{3}$$

The kinetic energy is

$$T = \frac{1}{2} I_3 \dot{\theta}^2 = \frac{1}{12} M a^2 \dot{\theta}^2 \tag{4}$$

and the potential energy is

$$U = \frac{M g a}{2\sqrt{3}} (1 - \cos \theta) \tag{5}$$

Therefore,

$$L = \frac{1}{12} M a^2 \dot{\theta}^2 + \frac{M g a}{2\sqrt{3}} \cos \theta \tag{6}$$

where the constant term has been suppressed. The Lagrange equation for θ is

$$\ddot{\theta} + \sqrt{3} \frac{g}{a} \sin \theta = 0 \tag{7}$$

and for oscillations with small amplitude, the frequency is

$$\boxed{\omega = \sqrt{\sqrt{3} \frac{g}{a}}} \tag{8}$$