11-13. We get the elements of the inertia tensor from Eq. 11.13a:

$$I_{11} = \sum_\alpha m_\alpha \left(x_{\alpha,2}^2 + x_{\alpha,3}^2 \right)$$

$$= 3m\left(b^2\right) + 4m\left(2b^2\right) + 2m\left(b^2\right) = 13mb^2$$

Likewise $I_{22} = 16mb^2$ and $I_{33} = 15mb^2$

$$I_{12} = I_{21} = -\sum_\alpha m_\alpha \, x_{\alpha,1} \, x_{\alpha,2}$$

$$= -4m\left(b^2\right) - 2m\left(-b^2\right) = -2mb^2$$

Likewise $I_{13} = I_{31} = mb^2$

and $I_{23} = I_{32} = 4mb^2$

Thus the inertia tensor is

$$\boxed{\{I\} = mb^2 \begin{bmatrix} 13 & -2 & 1 \\ -2 & 16 & 4 \\ 1 & 4 & 15 \end{bmatrix}}$$

The principal moments of inertia are gotten by solving

$$mb^2 \begin{vmatrix} 13-\lambda & -2 & 1 \\ -2 & 16-\lambda & 4 \\ 1 & 4 & 15-\lambda \end{vmatrix} = 0$$

Expanding the determinant gives a cubic equation in λ:

$$\lambda^3 - 44\lambda^2 + 622\lambda - 2820 = 0$$

Solving numerically gives

$$\lambda_1 = 10.00$$

$$\lambda_2 = 14.35$$

$$\lambda_3 = 19.65$$

$$\boxed{\begin{array}{c} \text{Thus the principal moments of inertia are } I_1 = 10\, mb^2 \\ I_2 = 14.35\, mb^2 \\ I_3 = 19.65\, mb^2 \end{array}}$$

To find the principal axes, we substitute into (see example 11.3):

$$(13-\lambda_i)\,\omega_{1i} - 2\omega_{2i} + \omega_{3i} = 0$$

$$-2\,\omega_{1i} + (16-\lambda_i)\,\omega_{2i} + 4\omega_{3i} = 0$$

$$\omega_{1i} + 4\omega_{2i} + (15-\lambda_i)\,\omega_{3i} = 0$$

For $i = 1$, we have $(\lambda_1 = 10)$

$$3\omega_{11} - 2\omega_{21} + \omega_{31} = 0$$

$$-2\omega_{11} - 6\omega_{21} + 4\omega_{31} = 0$$

$$\omega_{11} - 4\omega_{21} + 5\omega_{31} = 0$$

Solving the first for ω_{31} and substituting into the second gives

$$\omega_{11} = \omega_{21}$$

Substituting into the third now gives

$$\omega_{31} = -\omega_{21}$$

or

$$\omega_{11} : \omega_{21} : \omega_{31} = 1 : 1 : -1$$

So, the principal axis associated with I_1 is

$$\boxed{\frac{1}{\sqrt{3}}(x + y - z)}$$

Proceeding in the same way gives the other two principal axes:

$$\boxed{\begin{array}{ll} i = 2: & -.81\mathbf{x} + .29\mathbf{y} - .52\mathbf{z} \\ i = 3: & -.14\mathbf{x} + .77\mathbf{y} + .63\mathbf{z} \end{array}}$$

We note that the principal axes are mutually orthogonal, as they must be.