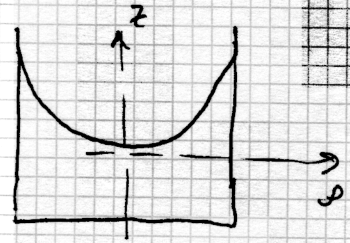


The surface of the fluid is perpendicular to the effective gravitational field.
The effective gravitational acceleration is

$$\vec{g}_{eff} = \vec{g}_0 + \vec{g}_{centrifugal}$$

$$\vec{g}_0 = -g_0\hat{z}$$

$$\vec{g}_{cent} = \omega^2 \rho\,\hat{\rho}$$

[see fig 10-6]

$-\vec{g}_{eff} = -g_0\hat{z} + \omega^2\rho\hat{\rho}$. The surface perpendicular to this is an equipotential, so we can use techniques from the vector calc. chapter to obtain the potential and set it equal to a constant.

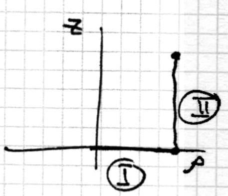
$$\frac{\vec{F}}{m} = -\vec{\nabla}\frac{U}{m}$$

[can check that this corresponds to a conservative force field: $\vec{\nabla}\times\vec{g} = 0$, by eq. F.11]

$$-\vec{g}_{eff} = -\vec{\nabla}\frac{U}{m}$$

$$g_0\hat{z} - \omega^2\rho\hat{\rho} = \vec{\nabla}\frac{U}{m}$$

Since we can choose any path to calculate $\frac{U}{m}(\rho, z)$, let's keep it simple:



$$\frac{U}{m} = \int_{z=0,\,\rho=0}^{z=0,\,\rho=\rho} \underbrace{(g_0\hat{z} - \omega^2\rho\hat{\rho})\cdot d\vec{l}}_{d\vec{l} = \hat{\rho}\,d\rho} + \int_{z=0,\,\rho=\rho}^{(z,\rho)} \underbrace{(g_0\hat{z} - \omega^2\rho\hat{\rho})\cdot d\vec{l}}_{\hat{z}\,dz}$$

$$= \int_{(0,0)}^{(0,\rho)} -\omega^2\rho\,d\rho + \int_{(0,\rho)}^{(z,\rho)} g_0\,dz = -\frac{\omega^2\rho^2}{2} + g_0 z$$

so: $\frac{U}{m} = z\cdot g_0 - \frac{\omega^2\rho^2}{2} = \text{const}$ is the curve that follows the surface of the water in the bucket.

or, if the center is at $z = \rho = 0$, this is

$$\boxed{z = \frac{\omega^2}{2g}\rho^2}$$, a parabola