Physics 43, Fall 2005

Exam 2

November 29, 2005

Five problems, three hours. Show your work, draw the appropriate diagrams, and explain in words the logic behind your calculations. Don't make me guess what you intend. If you have questions about what I intend in a problem, ask me rather than guess what I mean.

Weaseling out of things is important to learn. It's what separates us from the animals ... except the weasel.

A damped harmonic oscillator (mass m, spring constant k, damping constant b) is driven by a periodic rectangular force pulse. The pulses have amplitude f, each has a duration $\Delta\tau$, and they repeat with period τ.

1. Find the Fourier series of the periodic force. [I like to use the exponential version of the series, but you can use trigonometric functions if you wish.]
2. Find the long-time response of the harmonic oscillator (i.e. the particular solution $x_p(t)$).

I'm better than dirt. Well, most kinds of dirt, not that fancy store-bought dirt... I can't compete with that stuff.

Find the geodesics on a cone whose equation in cylindrical polar coordinates is $z = \lambda\rho$, where λ is a constant. [Let the required curve have the form $\phi = \phi(\rho)$].

I've been muscled out of everything I've ever done, including my muscle-for-hire business.

A simple pendulum (mass m, length l) whose point of support P is attached to the edge of a wheel (center O, radius R) that is forced to rotate at a fixed angular velocity ω. At $t = 0$, the point P is level with O on the right. Write down the Lagrangian and find the equation of motion for the angle ϕ. [See Figure.]

Nacho, nacho man. I want to be a nacho man...

Find the gravitational self-energy (energy of assembly piecewise from infinity) of a sphere of mass M and radius R.

Lisa, remember me as I am - filled with murderous rage.

Consider a pair of particles, each of mass m, which interact via a potential energy $U = kr^n$.

1. What does the condition $kn > 0$ tell us about the force?

2. Sketch the effective potential for $n = 2, -1, -3$.
3. Find the radius at which the particles (with fixed angular momentum ℓ) can orbit at a fixed radius.
4. For which values of n is the orbit stable?
5. For the stable case, show that the period of small oscillations about the circular orbit is $\tau_{osc} = \tau_{orb}/\sqrt{n+2}$