Physics 43, Fall 2004

Exam 2

November 29, 2004

famine A solid homogeneous cylinder of radius r and mass m rolls without slipping on the inside of a stationary larger cylinder of radius R as shown in Fig. 1.

1. Express the condition for rolling without slipping in terms of θ, the angle that the center of the small cylinder makes with the horizontal, and φ, the angle about its own axis through which the small cylinder has rolled.
2. Write down the Lagrangian for the system and determine the equation of motion of the inside cylinder.
3. Find the period of small oscillations about the stable equilibrium.

war A merry-go-round starts from rest and accelerates with a constant angular acceleration of 0.02 revolutions per second per second. A student sits in a chair $6m$ from the axis of revolution and holds a $2kg$ ball.

1. Derive the expression for the force on the ball in the rotating frame.
2. Calculate the magnitude and direction of the force $5s$ after the merry-go-round starts to rotate.

pestilence Consider a planet of mass m in orbit around a sun of mass M. Assume further there is a uniform distribution of dust, of density ρ, throughout the space surrounding the sun and the planet.

1. Show that the effect of the dust is to add an additional attractive central force $F' = mkr$ (neglect any drag force due to collision with dust particles), and find k in terms of ρ and G.
2. Write down the Lagrangian and obtain the equations of motion.
3. Sketch the effective potential for the effective $1D$ problem.
4. Consider a circular orbit for the planet corresponding to angular momentum L. Give an equation for the radius r_0 of the orbit in terms of L, M, m, and k (you need not solve it).
5. Assume F' is small in comparison with the attraction due to the sun, and consider an orbit deviating just slightly from the circular orbit of the previous part. By considering the frequencies of the radial and azimuthal motion, show that the orbit is a precessing ellipse and calculate the angular frequency of precession in terms of r_0, ρ, G, and M.

6. Does the ellipse precess in the same direction as the angular velocity or the oppositie direction?

death A long inextensible cable (or chain) of length L and weight per unit length W is suspended from posts separated by a distance $H < L$. Obtain an expression for the curve of the cable (you may express the curve parametrically in terms of the arc length s, i.e. as $y(s)$ and $x(s)$, if you wish).

bob What are Kepler's three laws?