Physics 43, Fall 2004

Exam 1

October 19, 2004

1. A particle of mass m is constrained to move on the frictionless inner surface of a cone of half-angle α under the influence of gravity, as shown in the figure.
   (a) Express Newton's second law in spherical coordinates (i.e. write $F_r = \ldots$, $F_\theta = \ldots$, etc.)
   (b) Find the restrictions on the initial conditions such that the particle moves in a circular orbit about the vertical axis.
   (c) Determine whether this kind of orbit is stable.
2. A uniform plank of length $2a$ is held temporarily so that one end leans against a frictionless vertical wall and the other end rests on a frictionless floor making an angle $\theta = \theta_0$ with the floor. When the plank is released, it will slide down under the influence of gravity.
   (a) Find an expressions (as an integral, if you wish) for the time it takes the plank to reach a new angle θ.
   (b) At what value of θ will the upper end of the plank leave the wall?
3. A squirrel of mass m runs at a constant speed V_0 relative to a cylindrical exercise cage of radius R and moment of inertia I. The cage has a damping torque proportional to its angular velocity. Neglect the dimensions of the squirrel with respect to R. If initially the cage is as rest and the squirrel is at the bottom and running, find the motion of the squirrel relative to a fixed coordinate system in the small oscillation, underdamped case. Find the squirrel's angular velocity in terms of its angle relative to the vertical for arbitrary angular displacements for the underdamped case by proceeding as follows:
   (a) Write the equation of motion for the squirrel in terms of the squirrel's angle from the bottom (call it $\theta(t)$. Write the equation of motion for the cage in terms of the cage's angle of displacement (call it $\varphi(t)$).
   (b) Obtain a single equation for $\theta(t)$, and consider the small oscillation approximation.
   (c) Solve that equation.
4. A chain with mass/length=u hanging vertically from one end, where an upward force F (which is insufficient to overcome the force of gravity) is applied to it, is lowered onto a table. Find the equation of motion for h, the height of the end above the table (h is the length of chain hanging freely).

5. A mass m on a spring with spring constant k and linear damping constant b is driven by a periodic force pulse with period τ and width $\Delta\tau$ and amplitude F_0 (i.e. $F(t) = F_0$ for $0 \leq t < \Delta\tau$, $F(t) = 0$ for $\Delta\tau < t < \tau$, repeated with period τ). What is $x(t)$?