Problem Set 9

Due: Tuesday, Apr. 22, 2003, at 11:59 pm.

Reading: Pain, Chap 5: pp. 122-130; Chap. 6, pp. 159-169, 171-173.

1. Pain, 6.1.
2. Pain, 6.3.
4. Pain, 6.7.
5. Pain, 6.16.

6. Show that, without any approximations, $\rho(x, t)$ satisfied the equation

$$\frac{\partial^2}{\partial t^2} \left( \frac{1}{\rho} \right) = \frac{\partial}{\partial x} \left[ c^2 \frac{\partial}{\partial x} \left( \frac{1}{\rho} \right) \right],$$

where

$$c^2(\rho) = P'(\rho) \left( \frac{\rho}{\rho_0} \right)^2.$$

Likewise, show that

$$\frac{\partial}{\partial t} \left[ \frac{\partial P}{\partial t} \left( \frac{1}{c^2} \right) \right] = \frac{\partial^2 P}{\partial x^2},$$

where $c^2$ is the quantity $P'(\rho)(\rho/\rho_0)^2$ expressed as a function of $P$.

7. The pressure amplitude of a fairly intense sound wave in air is the order of $2N/m^2$. Calculate the amplitudes of $\xi$, $\frac{\partial \xi}{\partial t}$, and $s$ for a sinusoidal wave of frequency 1kHz. How much above room temperature is the temperature at the center of a condensation?

8. Consider a wave which is initially sinusoidal, having a condensation amplitude $s_m$. Show that if $s_m$ is small but not entirely negligible, the expression for the effective velocity can be expanded to yield $c_{\text{max}} = c_0[1 + (1 + \gamma)s_m/2]$ for a crest and $c_{\text{min}} = c_0[1 - (1 + \gamma)s_m/2]$ for a trough. Each crest will tend to overtake the trough ahead of it. If we agree to call the wave distorted when the distance from crest to trough has been reduced to 9/10 of its original value, show that the distance that the wave must travel before it has this much distortion is given by

$$d = \frac{1}{20(\gamma + 1)s_m} \lambda.$$

Calculate numerical values from the data of the preceding problem.
9. Consider a right-moving acoustic wave which has a $t = 0$ displacement profile that is an isosceles triangle of width 2$a$ and height $b$, and whose leftmost point is at $x = 0$. Sketch the $t = 0$ profiles of $\xi$, $\partial \xi / \partial t$, $s$, and $p$, labelling the appropriate magnitudes.

10. Consider an acoustic disturbance possessing spherical symmetry in which the only displacement of the particles is in the radial direction. Let this by symbolized in Lagranian notation by $\xi(r, t)$. Set up the equations governing the motion of the displaced volume element. Make suitable approximations and obtain an equation in $p$ as the only dependent variable. By suitable rearrangement this equation can be put in the form of the regular one-dimensional wave equation. Obtain from this the general solution to the original problem.