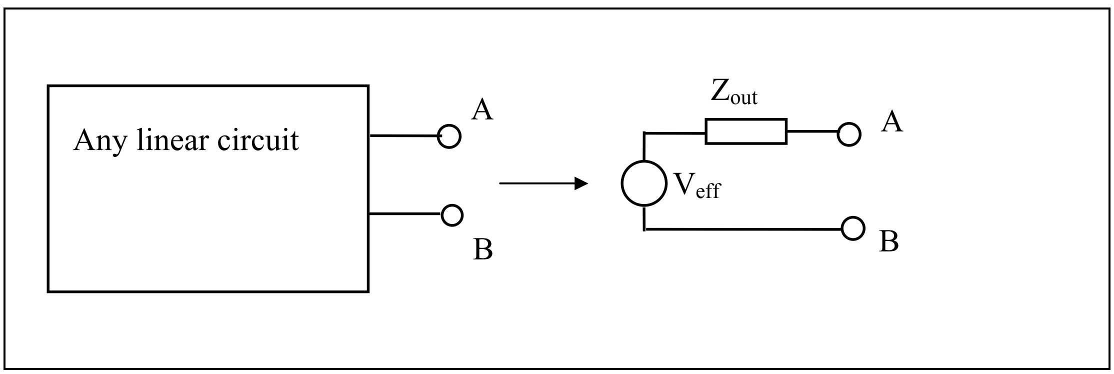
Input and Output Impedance and Thevenin's Theorem

I. Thevenin's Theorem

These notes discuss an amazing and important property of **linear** electrical circuits: If the circuit is connected to the outside world by two wires as shown below and if the circuit consists purely of linear circuit elements (fixed emf sources of a single frequency, resistors, inductors, and capacitors), Thevenin's Theorem states that the circuit, no matter how complicated, behaves electrically like an emf source in series with a fixed impedance. (Alternatively, the circuit can be replaced by a fixed current source with a fixed impedance in parallel. But for most cases, the fixed emf source model is more useful.) This theorem gives us a tremendous simplification in analyzing how our circuit will behave if we connect it to other electrical circuits.



If the circuit is viewed as providing current or voltage to a subsequent stage, the impedance is called the output impedance. (You may have noted that many stereo amplifiers list an **output impedance** of 8 ohms or so.) If the leads are viewed as an input for other signals, the impedance is called the **input impedance**.

II. Determining the Equivalent Circuit EMF and Impedance

It is relatively easy to determine the emf and impedance of the equivalent circuit. First, let's look at the output-type equivalent circuit. The effective emf can be determined by measuring the voltage between the two terminals with a device whose own impedance is much larger than that of the equivalent circuit. In many cases an oscilloscope (with an input impedance of typically 1 Megohm) or a digital multimeter are fine. This voltage is sometimes called the **open circuit voltage** ("open" because essentially no current is drawn from the circuit).

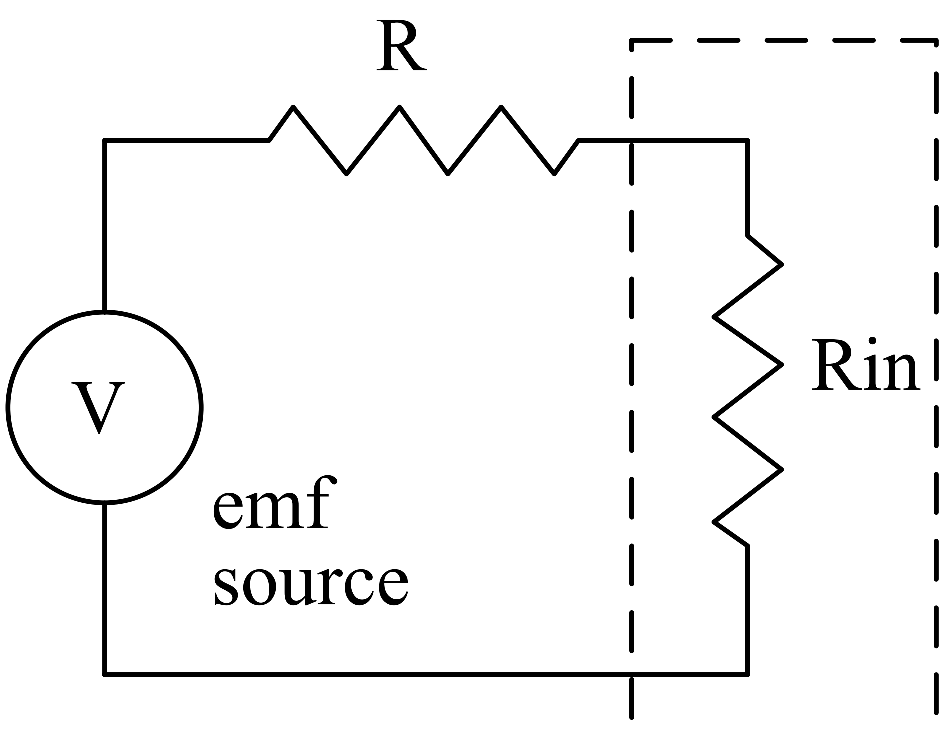
To determine the effective impedance is a bit more complicated. We will treat the case when the impedance is purely resistive. We connect a known resistance R across the output terminals (A and B in the figure above)and then measure the voltage across that resistor (again using a high impedance device like an oscilloscope). The voltage across R is easily seen to be

$$V_R = \frac{V_{eff} R}{R + R_{out}} \tag{1}$$

where V_{eff} is the equivalent emf of the circuit, either the DC voltage, or the AC amplitude. From Eq. (1), we can easily find R_{out}. **A Useful Trick**: If you have a variable resistor R, you can adjust R until $V_R = 0.5\ V_{\text{eff}}$. Then $R = R_{\text{eff}}$.

To determine an input impedance, when the circuit does not normally have an internal emf source, you can apply an emf source in series with a known resistance R. By measuring the voltage across the input terminals and comparing that value to the original emf value, you can determine R_{in}.

This procedure often runs into difficulty if R_{in} is large, say greater than 1 Megohm, because it may be difficult to find a measuring device with a sufficiently large impedance. (What goes wrong if $R_{\text{device}} < R_{\text{in}}$?)



If the circuit being tested also has an output, for example, if the circuit is an amplifier or an oscilloscope or a digital meter, we can use the device output as a monitor of the voltage across R_{in}. If R is adjustable, we will have the voltage across R_{in} equal to half V when $R = R_{\text{in}}$.

III. Why is Output Impedance Important?

The output impedance of a device can have a major effect on the signal that it transmits to the next stage of your circuitry. Suppose that the next stage has an input resistance R. Then the voltage that gets to that next stage is just the voltage V_R given in Eq. (1). It is easy to see that V_R is necessarily smaller than V_{eff}.

If we are concerned with transferring electrical power to the next stage (as we would be if our device is an amplifier driving some loudspeakers), then we get the maximum power developed in the output "load" (assuming a fixed V_{eff} and a fixed R_{out}) when $R_{load} = R_{out}$.

Exercise: Prove the assertion of the previous paragraph.

IV. Some Measurements of Input and Output Impedances.

A. Using the ideas discussed above, design and carry out a procedure to measure the output resistance of an old battery (to be provided). (In some texts, the output resistance of a battery is called its "internal resistance.")

B. Apply the same ideas to determine the output resistance of your function generator. Is that output resistance the same for both of the output switch settings (2 volts and 20 volts)?

C. Devise and carry out a procedure to determine the input resistance of the circuit contained in the circuit boxes provided for a signal of 1 kHz. (Since we have an ac signal you cannot simply use an ohmmeter to measure the resistance between the two input connections.)

D. Devise and carry out a procedure to determine the input resistance of your oscilloscope. Since the last paragraph of Section III for a hint.

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