Lab 7B: Fourier Synthesis and Analysis

One of the most powerful notions in all of physics and mathematics is that a complicated mathematical function can be represented as a sum (superposition) of simpler mathematical functions. One of the most common decompositions is that introduced by Fourier, in which the complicated mathematical function is given as a sum of simple sine and cosine (or exponential) functions. In this lab we explore some simple features of Fourier synthesis (finding the combination of sine and cosine functions that account for a particular complicated function).

Before coming to lab: Find (either calculated yourself or find in a book or your notes) the Fourier decomposition of a periodic square wave (with period $T_0$ and corresponding frequency $f_0$) and a periodic triangle wave. For each of these you will need to know the relative amplitude of each of the Fourier components (at $f_0$, $2f_0$, $3f_0$,...) and the phase for each of the components. (Note: since the amplitude of a complicated waveform can change without changing the Fourier decomposition except for an overall multiplicative factor, we usually give the magnitude of the various Fourier components relative to the magnitude for a fundamental component $f_0$).

Please bring a PC-formatted diskette to lab.

A. Fourier Synthesis

In this part of the lab you will use the Pasco Model 9307 Fourier Synthesizer to generate a variety of waveforms. You will observe the waveforms on an oscilloscope and listen to them with a loudspeaker.

1. Turn on the synthesizer and the oscilloscope. Note the various switches on the synthesizer that allow you to switch In and Out various sinusoidal waves whose frequencies are integer multiples of a fundamental frequency $f_0 \approx 437$Hz. For each harmonic you can adjust the amplitude and the phase using the controls on the synthesizer. The output of the synthesizer is also connected to a loudspeaker so that you can hear what the waveform sounds like.

2. Try out the various controls while observing the waveform on the oscilloscope. Be sure you understand how each control operates.

3. Now combine the fundamental frequency sine wave with the second harmonic of about the same amplitude as the fundamental. In musical terms, the second harmonic is one octave above the fundamental in pitch. You will notice that as you vary the phase of the second harmonic the waveform on the oscilloscope changes. Can you hear any difference in the sound as you change the phase? What does that tell you about the human auditory system?
An aside: Listen to the output as you rapidly turn the phase control for the second harmonic. Now you will probably hear something—something that sounds like a beat note. This occurs because a time-varying phase shift is equivalent to a frequency shift.

4. Using your calculations for the Fourier components of a periodic square wave, set the synthesizer to produce a waveform that should be fairly close to a periodic square wave. You will need to pay attention to both amplitude and phase. HINT: Use the synthesizer’s switches so that you can observe each harmonic on the oscilloscope individually. Use the oscilloscope display to help you adjust the amplitude. The phase can be set by using the controls on the synthesizer.

NOTE: Because we are using “only” nine harmonics, the square wave will not be perfect. It will exhibit some overshoot (or undershoot) at the temporal locations where the square wave is (ideally) discontinuous. This effect is known as the Gibbs phenomenon after J. Willard Gibbs, a Yale physicist who first recognized the effect about 100 years ago.