
7. Calculate the $n^{\text{th}}$ term in the Fibonacci sequence: $(1, 1, 2, 3, 5, \ldots, u_{n+2} = u_{n+1} + u_n, \ldots)$. To get started, write the sequence as a matrix equation:

$$\begin{pmatrix} u_{n+2} \\ u_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_{n+1} \\ u_n \end{pmatrix}.$$ 

So, if $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ the first few terms in the sequence can be found from

$$\begin{pmatrix} u_3 \\ u_2 \end{pmatrix} = A \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$\begin{pmatrix} u_4 \\ u_3 \end{pmatrix} = A^2 \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

and more generally,

$$\begin{pmatrix} u_n \\ u_{n-1} \end{pmatrix} = A^{n-2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Use what you know about matrices to find a closed-form expression for a general term in the series.